area $AaB+$area $PNMlp=$area $FGHKhg$. The intersections are $x=32^\circ\ 48'$, $x=61^\circ$, $x=119^\circ$, $x=147^\circ\ 12'$, $x=244^\circ\ 22'$, $x=295^\circ\ 38'$. The integrations are exceedingly tedious, but can be performed. If 3.1416 had been used for π the curves for one revolution would have consisted of four parts each equal to $aBCc$ and $ABRCD$.

MECHANICS.

Criticism on Professor Zerr's Solution of Problem 67, Mechanics, by J. M. ARNOLD, Crompton, R. I.

I wish to take exception to Professor Zerr's solution of No. 63 Mechanics, in the May number. The preliminary reasoning and the diagram are correct, but when he proceeds to find the required angles he commences with the assumption "The $\angle ABC=\angle CDE$ and the $\angle BAC=\angle CED$." This is wrong as it can be easily shown that these angles are not equal. Therefore his result must be in error. I have not had time to solve the problem correctly, but I think it leads to very complicated equations.

70. Proposed by CHARLES E. MEYERS, Canton, Ohio.

A homogeneous sphere, radius r, having an angular velocity ω, gradually contracts by cooling. What will be the angular velocity at the instant the radius becomes $\frac{1}{2}r$?

Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy. Ohio University, Athens, Ohio.

Let $m=$the constant mass ; r, $\frac{1}{2}r$ the original and final radii ; ω', the required angular velocity ; k, k' the radii of gyration corresponding.

The moment of angular momentum remaining constant,

$$mk^2\omega=mk'^2\omega' \ldots\ldots (1).$$

But $k^2=\frac{1}{2}r^2$, $k'^2=\frac{1}{2}(\frac{1}{2}r)^2=\frac{1}{8}r^2$, (1) plainly gives $\omega'=4\omega$.

Also solved in the same manner by *G. B. M. ZERR.*

71. Proposed by the late B. F. BURLESON, Oneida Castle, N. Y.

Three men own a sphere of gold the density of which varies as the square of the distance from the center. If two segments be cut off each one inch from the center of the sphere it will be divided into three parts of equal value. Determine the diameter of the sphere.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

Let $\rho=$density$=r^2$ in this case, $a=$radius. Then the mass of each segment cut off is

$$M=\int_1^a\int_0^{2\pi}\int_0^{\cos^{-1}(1/a)}\rho r^2\sin\theta d\theta d\varphi dr=\int_1^a\int_0^{2\pi}\int_0^{\cos^{-1}(1/a)}r^4\sin\theta dr d\varphi d\theta$$

$$=\frac{2\pi}{5a}(a^5-1)(a-1).$$

The mass of the remaining part is

$$M_1=2\int_0^1\int_0^{2\pi}\int_{\cos^{-1}(1/a)}^{\frac{1}{2}\pi} r^4\sin\theta dr d\varphi d\theta=\frac{4\pi}{5a}.$$

But $M=M_1$.

$\therefore \frac{4\pi}{5a}=\frac{2\pi}{5a}(a^6-a^5-a+1).$

$\therefore a^6-a^5-a-1=0.$

$\therefore a^3-(1/a^3)=a^2+(1/a^2)$, or $t^3-t^2+3t-2=0$ where $t=a-(1/a)$.

Let $t=s+\frac{1}{3}$, $\therefore s^3+\frac{8}{3}s=\frac{29}{27}$.

$\therefore s=.381891,\ t=.715224.$

$a=\frac{1}{2}[t+\sqrt{(4+t^2)}]=1.4196$ inches nearly.

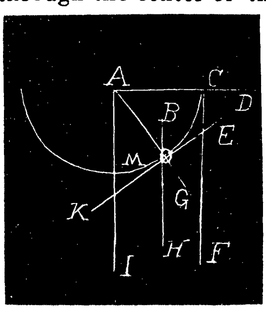
Similarly solved by *J. SCHEFFER*, his answer being 3.7462 inches.

72. Proposed by REV. A. L. GRIDLEY, Pastor of First Congregational Church, Kidder, Mo.

Prove that the motion of a ball falling through the earth influenced by gravity alone would be similar to the motion of a pendulum.

I. Solution by the PROPOSER.

It is an established fact that if a body should fall through an opening through the center of the earth, the net force drawing it toward the center is as the remaining distance to be traversed.



The pendulum ball follows the same law. It is evident that when the rod is horizontal all the force of gravity is exerted upon it to move it along its vertical tangent. When the rod is vertical, there is no net available force to move it from its position.

Making it general, by resolution of forces, the net force of gravity available to move the ball along the tangent EK is, in the figure, equal to the $\measuredangle AMB$. But this angle$=\measuredangle IAM$, as lines AI and BH are parallel and are cut by line AG.. But $\measuredangle IAG$ represents the remaining distance for the ball to pass through. Thus the law in both cases is the same.

Inference. 1st. To vibrate in the same time the whole arc—semi-circle—of the pendulum must equal the diameter of the earth.

2d. Vibrating past the center for any distance would require the same time as from surface to surface.

3d. The net available force acting is in inverse proportion to the distance passed through.

Similarly solved by *ALOIS F. KOVARIK*.